ΔT is the temperature difference between working surfaces.

Indices

e is the positive end;
p is the parasitic end;
0 is the initial value;
' is the concentration referred to region 1;
" is the concentration referred to region 2.

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DIFFERENTIAL TRANSFER EQUATIONS FOR MULTIPHASE, MULTICOMPONENT MEDIA

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Transfer equations of mass and momentum are obtained for single-phase, single-component and for multiphase, multicomponent media with account taken of substance change. Similarity criteria for these media are analyzed.

Investigations of transfer processes in multiphase, multicomponent media are topical problems in view of their wide application.

A considerable part of the investigations was extended in [1]. Further development was carried out in [2, 3]. In [2], transfer processes in a two-phase multicomponent medium are described and a thermodynamic analysis is carried out. The adopted assumptions, however, limit the range of applications for the obtained equations.

In the present article the transfer of mass and momentum in a multiphase, multicomponent (n, m) medium is described in accordance with the concepts of Sedov [4, 5], fruitfully applied by him to develop the mechanics of the multiphase media [1].

Let us consider a volume element of the medium with considerably smaller dimensions than those of the phase elements.

It is assumed that the transfer of a substance (mass, momentum) within a separate component, phase, or mixture can be described similarly as for a solid medium, but now the substance transport between the phases or components in this continuum is also taken into account.

In contrast to other investigations [2, 3], no restrictions are imposed as regards the effect of the shape, the number of phases (the number of phases $n \geq 1$), or the number of components (the number of components $m \geq 1$). The phases may be continuous or discrete. The elements of any phase may interact either with the

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elements of another phase or among themselves. It is assumed that chemical reactions only take place within a phase, that is, surface chemical reactions are regarded as leaking in direct contact though not on the interface boundary. Since various technical application changes of substances are of interest both for an individual component in an individual phase or medium and also for individual phases or media as entities, transport differential equations are deduced for each case.

1. Continuity Equations

1.1. For the transfer of the k-th component of the i-th phase of a multiphase flow one can write:

the flow rate of the mass of the k-th component of the i-th phase across the surface S of the volume V,

$$\int_S (\beta_i \mathbf{j}_{ik}^m \delta_p)\, dS;$$

the mass inflow of the k-th component of the i-th phase from other phases in the volume V,

$$-\int_V J_{ii-k}^m dV;$$

the mass inflow for the k-th component with transport of other components into the i-th phase:

$$-\int_V J_{iik-}^m\, dV.$$

The density change is

$$\frac{d}{d\tau}\int_V \beta_i \rho_{ik} dV.$$

The law of substance conservation is

$$\frac{d}{d\tau}\int_V \beta_i \rho_{ik}\, dV + \int_S (\beta_i\, \mathbf{j}_{ik}^m \delta_p)\, dS - \int_V (J_{ikk-}^m + J_{ii-k}^m) = 0. \tag{1}$$

The Leibnitz formula for the total derivative is applied,

$$\frac{d}{d\tau}\int_V \beta_i \rho_{ik} dV = \int_V \frac{\partial \beta_i \rho_{ik}}{\partial \tau}\, dV + \int_S \beta_i \rho_{ik} (\mathbf{v}_i \delta_p)\, dS; \tag{2}$$

then

$$\int_V \frac{\partial}{\partial \tau}\, \beta_i \beta_{ik} dV + \int_S \beta_i \rho_{ik} (\mathbf{v}_i \delta_p)\, dS + \int_S (\beta_i \mathbf{j}_{ik}^m \delta_p)\, dS - \int_V (J_{ikk-}^m + J_{ii-k}^m)\, dV = 0. \tag{3}$$

By applying the Ostrogradskii–Gauss formula,

$$\int_S (\mathbf{a}\delta_p)\, dS = \int_V (\nabla \cdot \mathbf{a})\, dV, \tag{4}$$

one obtains

$$\int_V \left\{\frac{\partial}{\partial \tau}\, \beta_i \rho_{ik} + \nabla [\beta_i (\rho_{ik} \mathbf{v}_i + j_{ik}^m] - J_{ikk}^m - J_{ii-k}^m \right\} dV = 0. \tag{5}$$

The continuity equation for the k-th component of the i-th phase is

$$\frac{d}{d\tau}\, \beta_i \rho_{ik} + \beta_i \rho_{ik} \nabla \cdot \mathbf{v}_{ik} - J_{ikk-}^m - J_{ii-k}^m = 0. \tag{6}$$

The expression for the mass source for the k-th component with interaction with the remaining components of the i-th phase (J_{ikk}^m) can be determined from the relations describing chemical reactions in a homogeneous medium. The mass of the k-th component which has passed across the phase boundary J_{ii-k}^m is represented by the sum of masses of the phase transition, coalescence, or disintegration. When analyzing processes in a specified motion [6] the relations for determining J^m_{il-k} must be suitably selected in each individual case.

1.2. For the transport of the k-th component in the (n, m) flux with $\sum_i J^m_{ii-k} = 0$ and $\sum_i J^m_{ikk-} = J^m_{kk-}$ one obtains in a similar way

$$\sum_i \left\{ \frac{d}{d\tau} (\beta_i \rho_{ik}) + \beta_i \rho_{ik} \nabla \cdot \mathbf{v}_i + \beta_i \, j^m_{ik} \right\} - J^m_{kk-} = 0. \tag{7}$$

If one sets

$$\frac{\rho_{ik}}{\rho_k} = \psi_{ik,k}, \quad \sum_i \beta_i \psi_{ik,k} = 1,$$

$$\sum_i \beta_i \rho_{ik} \mathbf{v}_{ik} = \rho_k \mathbf{v}_k, \quad \sum_i \beta_i \psi_{ik,k} \frac{v_{ik}}{v_k} = 1,$$

one obtains

$$\frac{\partial}{\partial \tau} \rho_k + \nabla \cdot \rho_k \mathbf{v}_k - J^m_{kk-} = 0. \tag{8}$$

Equation (8) has a formal character, whereas (7) reflects the complex character of the transfer of the k-th component in the multivelocity continuum.

1.3. For the transfer of the i-th phase* in a multiphase flux with $\sum_k J^m_{ik,k-} = 0$, $\sum_k J^m_{ii-k} = J^m_{ii-}$, and $\sum_k \psi_{ik,i} = 1$, where $\psi_{ik,i} = \rho_{ik}/\rho_i$, one obtains

$$\frac{d}{d\tau} \beta_i \rho_i + \beta_i \rho_i \nabla \cdot \mathbf{v}_i - J^m_{ii-} = 0. \tag{9}$$

Consequently, when determining the mass transfer of each phase in the medium, the transfer across the interface of all phase components must also be taken into account.

1.4. For multiphase flux one has

$$\sum_i \left\{ \frac{d}{d\tau} \beta_i \rho_i + \beta_i \rho_i \nabla \cdot \mathbf{v}_i \right\} = 0, \tag{10}$$

and denoting $\rho_k/\rho = \varphi_k$, $\sum_k \varphi_k = 1$,

$$\sum_k \rho_k \mathbf{v}_k = \sum_k \sum_i \beta_i \rho_{ik} \mathbf{v}_{ik} = \sum_i \beta_i \rho_i \mathbf{v}_i = \rho \mathbf{v},$$

*If the elements of the i-th phase differ in their geometric or physical characteristics, one has to describe the transfer processes for each j-th group of identical elements of the i-th phase. If there are p such groups, then there are p continuity equations:

$$\frac{d}{d\tau} \beta_{ij} \rho_{ij} + \beta_{ij} \rho_{ij} \nabla \cdot \mathbf{v}_{ij} - J^m_{ijj-} - J^m_{ii-j} = 0.$$

one has

$$\frac{\partial\rho}{\partial\tau}+\nabla\cdot\rho\mathbf{v}=0. \quad (11)$$

2. Equation of Moments

2.1. We obtain the equation of momenta for the k-th component of the i-th phase of the medium. The momentum from the k-th component of the i-th phase across the surface S of volume V is

$$\int_S [\beta_i\rho_{ik}\mathbf{v}_{ik}\delta_{\mathrm{p}}]\,dS.$$

The effect of the external surface forces on the k-th component of the i-th phase in the volume V is

$$\beta_i\int_S [P_{ik}\delta_{\mathrm{m}}]\,dS.$$

The effect of the external mass forces on the k-th component of the i-th phase in the volume V is

$$\int_V [\beta_i\rho_{ik}\mathbf{F}_{ik}]\,dV.$$

An increment of momentum of the k-th component of the i-th phase with mass and momentum inflow from other components of the i-th phase is

$$\int_V (-\mathbf{J}^q_{ikk^-})\,dV.$$

An increment of momentum of the k-th component of the i-th phase with mass and momentum from other phases in the volume V is

$$\int_V (-\mathbf{J}^q_{ii^-k})\,dV.$$

An increment of momentum according to the conservation law of substance is

$$\frac{d}{d\tau}\int_V \beta_i\rho_{ik}\mathbf{v}_{ik}dV+\int_S \beta_i\rho_{ik}\mathbf{v}_{ik}\delta_{\mathrm{p}}dS+\int_V(\beta_i\rho_{ik}\mathbf{F}_{ik}+\mathbf{J}^q_{ikk^-}+\mathbf{J}^q_{ii^-k})\,dV+\beta_i\int_S P_{ik}\delta_{\mathrm{p}}dS=0. \quad (12)$$

One finds after transformations similar to those in Sec. 1.1,

$$\beta_i\rho_{ik}\frac{d}{d\tau}\mathbf{v}_{ik}+\beta_i\nabla p_{ik}+\beta_i\nabla\cdot\sigma_{ik}-\beta_i\rho_{ik}\mathbf{F}_{ik}+(J^m_{ikk^-}+J^m_{ii^-k})\,\mathbf{v}_{ik}-\mathbf{J}^q_{ikk^-}-\mathbf{J}^q_{ii^-k}=0. \quad (13)$$

2.2. Similarly for

$$\sum_i \mathbf{J}^q_{ikk^-}=\mathbf{J}^q_{kk^-},\quad \sum_i \mathbf{J}^q_{ii^-k}=0$$

the differential equation for the momentum of the k-th component of a multiphase, multicomponent flow is

$$\sum_i\left\{\beta_i\rho_{ik}\frac{d}{d\tau}\mathbf{v}_{ik}+\beta_i\nabla p_{ik}+\beta_i\nabla\cdot\sigma_{ik}-\beta_i\rho_{ik}\mathbf{F}_{ik}+(J^m_{ikk^-}+J^m_{ii^-k})\,\mathbf{v}_{ik}\right\}-\mathbf{J}^q_{kk^-}=0. \quad (14)$$

If one sets $\mathbf{v}_{ik}=\mathbf{v}_k+\mathbf{v}_{\Delta ik}$, $\sum_i \beta_i\nabla p_{ik}=\nabla p_k$,

$$\sum_i \beta_i\nabla\cdot\sigma_{ik}=\nabla\cdot\sigma_k,\quad \sum_i \beta_i\rho_{ik}\mathbf{F}_{ik}=\rho_k\mathbf{F}_k$$

and bearing in mind that

$$\sum_i \beta_i\rho_{ik}\mathbf{v}_{ik}=\rho_k\mathbf{v}_k,\qquad \sum_i \beta_i\rho_{ik}\mathbf{v}_{\Delta ik}=0,$$

one obtains

$$\rho_k \frac{\partial}{\partial\tau} \mathbf{v}_k + \rho_k \mathbf{v}_k \cdot \nabla \mathbf{v}_k + \nabla p_k + \nabla \cdot \sigma_k - \rho_k \mathbf{F}_k + J^m_{kk-} \mathbf{v}_k - \mathbf{J}^q_{kk-} + \mathbf{J}^q_{k\Delta ik} = 0. \quad (15)$$

In the above

$$\mathbf{J}^q_{k\Delta ik} = \sum_i \Big[\beta_i \rho_{ik} \frac{\partial}{\partial\tau} \mathbf{v}_{\Delta ik} + \beta_i \rho_{ik} \mathbf{v}_k \cdot \nabla \mathbf{v}_{\Delta ik} + \beta_i \rho_{ik} \mathbf{v}_{\Delta ik} \cdot \nabla \mathbf{v}_{\Delta ik} + \beta_i \rho_{ik} \mathbf{v}_{\Delta ik} \cdot \nabla \mathbf{v}_k + (J^m_{ikk-} + \mathbf{v}_{dik}) \Big]. \quad (16)$$

The last term of (16) reflects the difference in the transport of momentum of the k-th component in a multiphase medium as compared with the transfer in a single-phase medium.

2.3. For the equation of the momentum of the i-th phase with

$$\sum_k \mathbf{J}^q_{ikk-} = 0, \quad \sum_k \mathbf{J}^q_{ii-k} = \mathbf{J}^q_{ii-}, \quad \mathbf{v}_{ik} = \mathbf{v}_i + \mathbf{v}_{dik}, \quad \sum_k \rho_{ik} v_{dik} = 0,$$

$$\sum_k p_{ik} = p_i, \quad \sum_k \sigma_{ik} = \sigma_i, \quad \rho_i \mathbf{F}_i = \sum_k \rho_{ik} \mathbf{F}_{ik}$$

one obtains

$$\beta_i \rho_i \frac{d}{d\tau} \mathbf{v}_i + \beta_i \nabla p_i + \beta_i \nabla \cdot \sigma_i + J^m_{ii-} \mathbf{v}_i - J^q_{ii-} - \beta_i \rho_i \mathbf{F}_i + \sum_k (J^m_{ikk-} + J^m_{ii-k}) \mathbf{v}_{dik} = 0. \quad (17)$$

If in the particular case of the two-phase medium one neglects the pressure change as well as the dynamic and inertial effects of the diffusion rates, one obtains the equation of motion derived in [2].

2.4. For the equation of momenta of a multiphase, multicomponent flux with

$$\sum_i \mathbf{J}^q_{ii-} = 0, \quad \sum_i \Big[\beta_i \rho_i \frac{d}{d\tau} \mathbf{v}_i + \beta_i \nabla p_i + \beta_i \nabla \cdot \sigma_i + J^m_{ii-} \mathbf{v}_i - \beta_i \rho_i F_i + \sum_k (J^m_{ikk-} + J^m_{ii-k}) \mathbf{v}_{dik} \Big] = 0 \quad (18)$$

and if one sets

$$\mathbf{v}_i = \mathbf{v} + \mathbf{v}_{\Delta i}; \sum \beta_i \nabla p_i = \nabla p; \quad \sum \beta_i \rho_i \mathbf{F}_i = \rho \mathbf{F}; \quad \sum \beta_i \nabla \cdot \sigma_i = \nabla \cdot \sigma,$$

then

$$\rho \frac{\partial}{\partial\tau} \mathbf{v} + \rho \mathbf{v} \cdot \nabla \mathbf{v} + \nabla p + \nabla \cdot \sigma - \rho \mathbf{F} + \mathbf{J}^q_{\Delta i} = 0, \quad (19)$$

where

$$\mathbf{J}^q_{\Delta i} = \sum_i \Big[\beta_i \rho_i \frac{\partial}{\partial\tau} \mathbf{v}_{\Delta i} + \beta_i \rho_i \mathbf{v} \cdot \nabla \mathbf{v}_{\Delta i} + \beta_i \rho_i \mathbf{v}_{\Delta i} \cdot \nabla \mathbf{v}_{\Delta i} + \beta_i \rho_i \mathbf{v}_{\Delta i} \cdot \nabla \mathbf{v} - \sum_k (J^m_{ikk-} + J^m_{ii-k}) \mathbf{v}_{dik} \Big]. \quad (20)$$

The last term of (19), that is, $J^q_{\Delta i}$, reflects the dynamic and inertial, effects due to the relative velocities $V_{\Delta i}$ of the phases and the diffusion rates of the components V_{dik}.

Thus, the obtained equations of the transfer of mass and momentum together with the energy equations not discussed in the present article provide a system of equations describing the transfer processes in the (n, m) medium regarded as a continuum. This system can be completed by adding to it the thermodynamic relations of the phase state and of the components and the relations for determining the substance on the interface and in the phases. These relations may be quite different, since they are determined by the structure of the specific (n, m) medium.

In applying the above-derived transfer equations of mass and momentum to describe processes taking place in a specified medium one can ignore the corresponding terms; in this way one obtains equations which are of more specialized character, but can be fairly easily solved.

Using the transfer equations for mass and momentum in an (n, m) medium and the conditions of the identity of the equations for two similar processes, one can easily obtain criteria whose numerical values are equal.

1. Similarity Criteria for Mass Transfer

1.1'. For the k-th component in the i-th phase of the flow one has

$$\frac{v_{ik}\tau}{l} = \mathrm{Ho}_{ik};\quad \frac{J^m_{ikk^-}\, l}{\rho_{ik}v_{ik}} = M_{ikk^-,ik};\quad \frac{J^m_{ii^-k}\, l}{\rho_{ik}v_{ik}} = M_{ii^-k,ik}.$$

1.2'. For the k-th component of the i-th phase m-component flow one has

$$\frac{v_k\tau}{l} = \mathrm{Ho}_k;\quad \frac{J^m_{kk^-}\, l}{\rho_k v_k} = M_{kk^-,k}.$$

1.3'. For the i-th phase in the (n, m) flow one has

$$\frac{v_i\tau}{l} = \mathrm{Ho}_i;\quad \frac{J^m_{ii^-}\, l}{v_i\rho_i} = M_{ii^-,i}.$$

1.4'. For the (n, m) flow one has $v\tau/l$ = Ho.

In the four cases of mass transfer in the (n, m) medium under consideration the criterion known from the continuity equation of single-phase, single-component media assumes different values: in Sec. 1.1' Ho_{ik}^{-1} is the ratio of the change in time of the local density of the k-th component in the i-th phase to the density of the k-th component when the velocity is v_{ik}; Ho_k^{-1} in Sec. 1.2' is the same ratio for the k-th component in the medium for the velocity v_k; Ho_i^{-1} in Sec. 1.3' is for the i-th phase of the medium for v_i; and Ho in Sec. 1.4 in the medium for the velocity v.

The following relation exists between these criteria:

$$\mathrm{Ho} = \sum_i \beta_i\varphi_i \mathrm{Ho}_i = \sum_i \beta_i\varphi_i \Big(\sum_k \psi_{ik,i}\mathrm{Ho}_{ik}\Big) = \sum_k \varphi_k \mathrm{Ho}_k = \sum_k \varphi_i \Big(\sum_i \beta_i\psi_{ik,k}\mathrm{Ho}_{ik}\Big).$$

Thus, in the case of a multiphase, multicomponent medium the criteria Ho, Ho_i and Ho_k are derived from the criteria Ho_{ik}. In the case of a multicomponent medium, Ho is derived from Ho_k, but in the multiphase case it is derived from Ho_i.

The similarity of the transfer processes of mass for the k-th component and for the i-th phase in an (n, m) medium requires also the equality of the specific (for this case) dimensionless complexes $M_{ik,k^-,ik}$, $M_{ii^-,k,ik}$, $M_{ii^-,i}$, and $M_{kk^-,k}$ in addition to the numerical equality of the homochronicity criteria. They represent the ratios of the changes in the density of the components for the phase due to the existence of mass sources to their density with the corresponding velocity.

The similarity in the transfer of the k-th component in the i-th phase presumes that the equality of the values of the complexes M_{ikk^-} and $M_{ii^-,ik}$ was observed for mass sources with chemical reactions and for phase transitions, respectively. In the transfer of the i-th phase and of the k-th component in the medium it suffices to observe that $M_{ii^-,i}$ and $M_{kk^-,k}$ are equal. The relations which are operational here are

$$M_{kk^-,k} = \frac{\sum_i M_{ikk^-,ik}\psi_{ik,k}v_{ik}}{v_k},\quad M_{ii^-,i} = \frac{\sum_i M_{ii^-k,ik}\psi_{ik,i}v_{ik}}{v_i}.$$

2'. Criteria and Equations in Dimensionless Form for the Momentum (Time Integral of Force) of the Medium

2.1'. For the k-th component of the i-th phase the following is obtained:

$$\frac{v_{ik}\tau}{l} = \mathrm{Ho}_{ik},\quad \frac{J^q_{ii^-k}}{v^2_{ik}\rho_{ik}}\, l = D_{ii^-k,ik},$$

$$\frac{p_{ik}}{v^2_{ik}\rho_{ik}} = \mathrm{Eu}_{ik},\quad \frac{J^q_{ikk^-}}{v^2_{ik}\rho_{ik}}\, l = D_{ikk^-,ik},$$

$$\frac{\sigma_{ik}}{v_{ik}^2 \rho_{ik}} = \mathrm{Re}_{ik}^0 , \quad \frac{J_{ikk^-}^m}{v_{ik} \rho_{ik}} l = M_{ikk^-,ik} ,$$

$$\frac{F_{ik}}{v_{ik}^2} l = \mathrm{Fr}_{ik}, \quad \frac{J_{ii^-k}^m}{v_{ik} \rho_{ik}} l = M_{ii^-,k,ik} .$$

2.2'. For the k-th component of the (n, m) medium the following criteria are obtained:

$$\frac{v_k \tau}{l} = \mathrm{Ho}_k, \quad \frac{J_{kk}^q}{v_k^2 \rho_k} l = D_{kk^-,k} , \quad \frac{p_k}{v_k^2 \rho_k} = \mathrm{Eu}_k,$$

$$\frac{J_{kk^-}^m}{v_k \rho_k} l = M_{kk^-,k}, \quad \frac{\sigma_k}{v_k^2 \rho_k} = \mathrm{Re}_k^0 ,$$

$$\frac{F_k}{v_k^2} l = \mathrm{Fr}_k, \quad \frac{\Delta J_{k\Delta ik}^q}{v_k^2 \rho_k} l = D_{k\Delta ik,k} .$$

2.3'. For the i-th phase of the (n, m) medium the following criteria are obtained:

$$\frac{v_i \tau}{l} = \mathrm{Ho}_i, \quad \frac{J_{ii^-}^q}{v_i^2 \rho_i} l = D_{ii^-,i}, \quad \frac{p_i}{v_i^2 \rho_i} = \mathrm{Eu}_i,$$

$$\frac{J_{ii}^m}{v_i \rho_i} l = M_{ii^-,i} , \quad \frac{\sigma_i}{v_i^2 \rho_i} = \mathrm{Re}_i^0 ,$$

$$\frac{F_i}{v_i^2} l = \mathrm{Fr}_i, \quad \frac{J_{i\Delta ik}^q}{v_i \rho_i} = D_{i\Delta ik}.$$

2.4'. For the (n, m) medium one has the following criteria:

$$\frac{v\tau}{l} = \mathrm{Ho}, \quad \frac{p}{v^2\rho} = \mathrm{Eu}, \quad \frac{J_{\Delta i} l}{v^2 \rho} = D_{\Delta i},$$

$$\frac{\sigma}{v^2\rho} = \mathrm{Re}^0, \quad \frac{F}{v^2} l = \mathrm{Fr}.$$

The criteria Ho, Eu, Re^0, and Fr in the (n, m) medium are determined by the ratio of forces as in the case of a single-phase one-component medium. One has four different groups of criteria whose equality is a necessary condition for the similarity of the transfer processes of the momentum in two (n, m) media depending on whether the forces act on the k-th component of the i-th phase or medium, or on the i-th phase of the medium, or on the medium. The criteria referring to the momentum transfer by phase or by medium component or by the entire medium are derivatives of the criteria for the k-th component of the i-th phase in accordance with the relations

$$\mathrm{Eu} = \frac{1}{v^2} \sum_i \beta_i v_i^2 \varphi_i \mathrm{Eu}_i = \frac{1}{v^2} \sum_{ik} \beta_i v_{ik}^2 \psi_{ik,i} \varphi_i \mathrm{Eu}_{ik} = \frac{1}{v^2} \sum_k v_k^2 \varphi_k \mathrm{Eu}_k .$$

One has similar relations for Re^0 and Fr.

Side by side with these criteria and the complexes of sources of mass, the similarity of the transfer of momentum in two (n, m) media also requires the maintenance of the equality of the dimensionless complexes $D_{ikk^-,ik}$, $D_{ii^-,k,ik}$, $D_{kk^-,k}$, and $D_{ii^-,i,k}$. They represent the ratio of the changes of local forces due to the presence of momentum sources with chemical reactions taking place and phase transitions to the forces at the corresponding velocities. For the latter one has the following relations:

$$D_{ii^-i} = \frac{1}{v_i^2} \sum_k \psi_{ik,i} v_{ik}^2 D_{ii^-,k,ik} , \quad D_{kk^-k} = \frac{1}{v_k^2} \sum_i \beta_i \psi_{ik,k} v_{ik}^2 D_{ikk^-,ik} .$$

The complexes $D_{k\Delta ik,k}$, $D_{i\Delta ik,i}$, and $D_{\Delta i}$ estimate the effect of diffusion rates and of heterogeneity of the (n, m) medium on the momentum transfer.

NOTATION

β is the true volume concentration;
ρ is the density;
P is the pressure of external forces;
p is the normal pressure;
σ is the tangential stress;
j is the flow density of substance;
J is the substance source;
F is the volume force.

Indices

i is the phase;
k is the component;
d is the diffusion;
m is the mass;
q is the momentum;
* is the dimensionless quantity.

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